

MIXED REAL/COMPLEX FACTORIZATION

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Abstract - This paper describes a mixed real/complex sparse matrix factorization and solution scheme applied to a large matrix problem. Large system eigenanalysis and frequency domain methods will directly benefit from the proposed scheme, which can reduce both memory and CPU time requirements when compared to conventional complex-only solutions. The application in hand is the small signal electromechanical stability analysis of large power systems. The savings obtained are significant considering the CPU intensive nature of these matrix problems.

Keywords - sparse matrix techniques, ordering algorithms, large eigenproblems, small signal stability, frequency domain analysis

I. INTRODUCTION

The use of sparse matrix and vector techniques to efficiently solve large simultaneous linear equations is the basis for almost all applications in power systems.

The basic approach, introduced in the sixties [1], is the sparsity preserving triangular decomposition of the coefficient matrix with the solution obtained via a forward and back substitution. Power system engineers have been responsible for a major part of the developments in the sparse matrix manipulation field [2]. Reference [3] provides an excellent and up-to-date review on power system sparsity techniques and a valuable list of references.

The efficiency of sparsity techniques is dependent on the skillful implementation of the algorithms and the exploitation of the coefficient matrix characteristics like symmetry, blocked structure, etc. Sparse matrix techniques need to be adapted to take full advantage of special characteristics of the matrix problems dealt with in different applications [4].

Network analysis deals mainly with structurally symmetric matrices, whose elements are either all real or all complex, and very good algorithms have been developed to handle these cases. This paper describes sparsity techniques developed to handle a different class of matrix problems which arise in the power system small signal stability area.

The power system stability problem needs to be formulated through the *augmented system equations* [5, 6, 7, 8, 9, 10, 11] for the efficient small signal analysis of large scale power systems. Eigenanalysis and frequency domain methods deal with a matrix problem of mixed nature, where a real coefficient matrix has complex elements added to just some of its diagonal positions.

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This fact can be exploited to advantage, resulting in the use of both real and complex arithmetic during the triangular decomposition and repeat solution. A tie-breaker criterion for the minimum-degree ordering [3] is used to minimize the complex arithmetic. A partial refactorization scheme [12] is also readily obtained from the knowledge of the complex pattern of the factors.

The basics of the proposed technique is described through use of the well-known rectangular shaped 20-bus system [3, 12, 13]. Results obtained for the 616-bus model of the Brazilian South-Southeast power system [10, 11] demonstrate the advantages in adopting the proposed technique.

These mixed real/complex techniques can be applied to matrix problems in other fields of engineering.

II. SMALL SIGNAL STABILITY BACKGROUND

The power system electromechanical stability problem can be represented by a set of differential equations, to be solved simultaneously:

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{r}) \\ \mathbf{0} &= g(\mathbf{x}, \mathbf{r}) \end{aligned} \quad (1)$$

where \mathbf{x} is the vector of the state variables and \mathbf{r} is the vector of algebraic variables.

Small signal stability analysis involves the linearization of (1) around a system operating point ($\mathbf{x}_0, \mathbf{r}_0$) resulting in the *augmented system equations* [5]:

$$\begin{pmatrix} \Delta \dot{\mathbf{x}} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{pmatrix} \begin{pmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{r} \end{pmatrix} \quad (2)$$

Figure 1 shows the Jacobian matrix topology for the well-known New England stability test system [5, 6, 11, 14] where the equations are organized in a different way to that shown in (2). The equations (differential and algebraic) related to each dynamic component of the system (synchronous machines, induction motors, static VAR compensators and HVDC links in this sequence) are conveniently assembled together, yielding the blocked diagonal form for this part of the matrix. The electrical network is modeled by algebraic equations expressed in the nodal admittance matrix form, which come last in the Jacobian matrix.

The matrix of Figure 1 has the *augmented system equations* arranged in the form:

$$\begin{pmatrix} \Delta \dot{\mathbf{x}}_a \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_A & \mathbf{J}_B \\ \mathbf{J}_C & \mathbf{J}_D \end{pmatrix} \begin{pmatrix} \Delta \mathbf{x}_a \\ \Delta \mathbf{y} \end{pmatrix} \quad (3)$$

where $\Delta \mathbf{x}_a$ is the augmented state vector, containing all the state and algebraic variables for the dynamic components and $\Delta \mathbf{y}$ is the